CS 214-02 Final Exam

Practice

Name _____

- The exam is worth 150 points total.
- You will have 2 hr 30 minutes for the exam.
- DON'T LEAVE ANY QUESTIONS BLANK. At least try.
- If you leave the classroom, you must turn in your exam.
- You may not use a calculator or notes.
- All answers must be written on the test paper.

1. Construct a truth table for the following propositional wff. Tell whether it is a tautology, contradiction, or neither.

   $(A \vee B) \wedge (C \rightarrow B')$

2. Given the truth values A true, B true, and C false, what is the truth value of each of the following wffs?

   a. $(A \wedge B) \rightarrow C$

   b. $(A \vee B) \wedge (C \rightarrow B')$

3. Use the first principle of mathematical induction to prove the following formula:

$$1\cdot 2+2\cdot 3+3\cdot 4+\cdots+n(n+1)=\frac{n(n+1)(n+2)}{3} \qquad n\geq 1$$

4. Let the sets
   $S = \{1, 3, 5, 7, 9\}$
   $T = \{0, 2, 4, 6, 8, 10\}$
   $U = \{0, 3, 6, 9\}$
   be subsets of the set $R = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

   a. List the elements of each of the following sets.

      i. $S \cap U$

      ii. S'

      iii. $T - U$

      iv. $(T - U) \times (S \cap U)$

   b. What is $|S|$? What is $|U|$? What is $|S \cup T|$?

5. Given events E_1 and E_2, with $P(E_1) = 0.4$, $P(E_2) = 0.25$, and $P(E_1 \cap E_2) = 0.15$, calculate the following.

   a. $P(E_1')$

   b. $P(E_1 \cup E_2)$.

6. For the equivalence relation
   $\rho = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2)\}$,
   what is the set [2]? List any other names for [2].

7. Identify each relation on N as one-to-one, one-to-many, many-to-one, or many-to-many.

   a. $\rho = \{(1, 2), (2, 3), (3, 3), (4, 1), (5, 6)\}$

   b. $\rho = \{(a, b), (a, d), (a, f), (b, c), (d, c)\}$

   c. $\rho = \{(2, 4), (4, 6), (6, 8), (8, 10), (10, 12)\}$

8. For the following functions from the domain to the codomain given, determine whether each function is onto, one-to-one, and/or a bijection.

   a. f: Z → N where f is defined by $f(x) = x^2 + 1$

   b. f: $\{1, 2, 3\} \rightarrow \{p, q, r\}$ where $f = \{(1, q), (2, r), (3, p)\}$

9. Given the function $f: S \rightarrow T$ described by $f = \{(6, 1), (0, 3), (4, 1), (1, 7), (2, 5)\}$, where $T = \{1, 2, 3, 5, 7\}$.

   a. List the elements in the domain of f.

   b. List the elements in the codomain of f.

   c. List the elements in the range of f.

   d. List the image(s) of 1.

   List the preimage(s) of 1.

10. Let f: N $\rightarrow$ N be defined by $f(x) = 2x$. Let g: N $\rightarrow$ N be defined by $g(x) = x + 1$. Calculate the following.

    a. $(g \circ f)(4)$

    b. $(f \circ g)(x)$

11. Let A = $\{a, b, c, d, e\}$. Write the following permutation function in cycle form and in array form.

    $g = \{(a, a\}, (b, c), (c, e), (d, d), (e, b)\}$

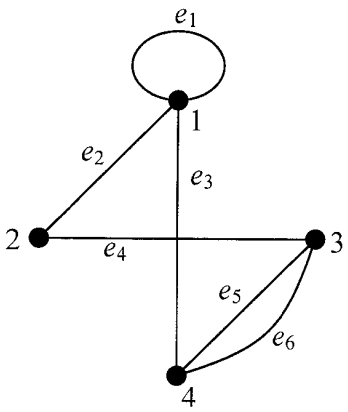
12. Given $r = 2$, $s = -2$, and

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \\ 3 & 4 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 2 & 1 \\ 3 & -1 \\ 1 & 0 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

Compute the following, if possible. If not possible, explain why:

a. $r\mathbf{A} + \mathbf{B}$
b. $\mathbf{A} \cdot \mathbf{B}$
c. $\mathbf{B} \cdot \mathbf{C}$
d. $s(\mathbf{B} - \mathbf{A})$
e. $s(\mathbf{A} + \mathbf{C})$

13. Consider the following graph



a. What is the degree of node 3? Node 1?

b. List any parallel arcs (edges).

c. List any loops.

d. List a cycle not involving e_1.

e. List 2 paths from node 2 to node 4.

f. Is the graph complete? Why or why not?

g. Is the graph simple? Why or why not?

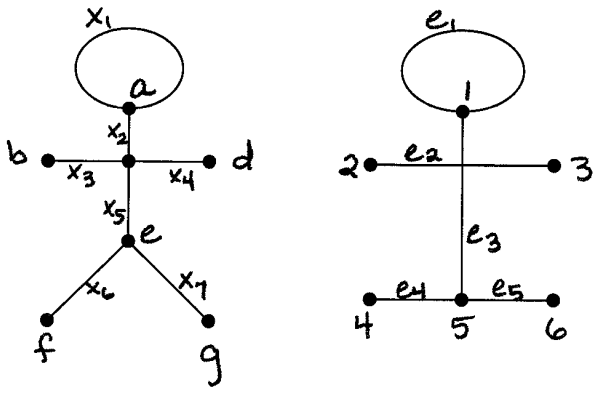
h. Is the graph connected? Why or why not?

14. Given the relation {(1, 4), (1, 2), (3, 2), (3, 1), (4, 3), (4, 4)}:
    a. Draw the directed graph that corresponds to the relation.

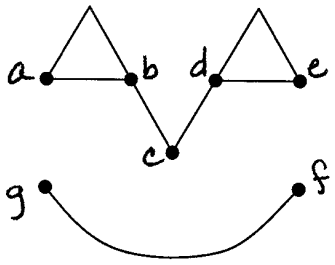
    b. Write the adjacency list that corresponds to the relation.

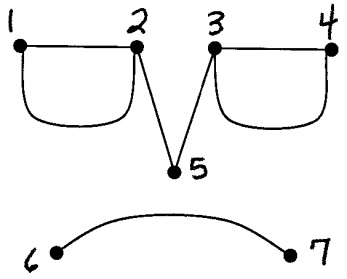
    c. Write the adjacency matrix that corresponds to the relation.

15. Determine whether the 2 graphs below are isomorphic. If so, give the function or functions that establish the isomorphism; If not, explain why not.



16. Determine whether the 2 graphs below are isomorphic. If so, give the function or functions that establish the isomorphism; If not, explain why not.



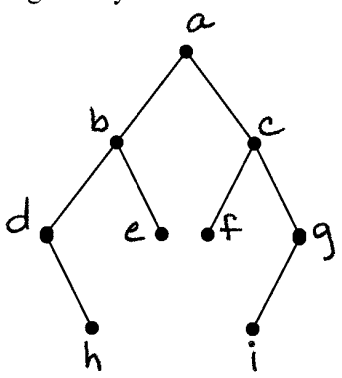


17. a. Draw an expression tree for the expression (5+3)-((4*6)/8)

b. Write the expression in prefix notation.

c. Write the expression in postfix notation.

18. Given the following binary tree:



a. List the elements in a preorder traversal.

b. List the elements in an inorder traversal.

c. List the elements in a postorder traversal.

19. Draw a decision tree for sequential search on 7 elements.

a. How many comparisons are done in the worst case?

b. How many comparisons are done if the item you are search for is the fourth item in the list?

c. How many comparisons are done in the worst case using sequential search on a list with n elements?

20. Draw the ***binary search tree*** for the following list of elements: r x y l s e h w a.

21. What is the depth of your tree in Problem 20?

22. Discuss the tradeoffs involved when choosing between an adjacency list and an adjacency matrix to represent a graph.

BONUS: What is the formula for the minimum number of comparisons that must be done in the worst case when searching a list of n elements?