CHAPTER 3

Section 3.4

Permutations & Combinations

- What is a permutation?
  - An ordered arrangement of objects.
- $P(n, r)$: the number of permutations of r distinct objects chosen from n distinct objects
- Formula: $$P(n,r) = \frac{n!}{(n-r)!}$$
- Example: Last 4 digits of a telephone number. Choose with no repeating digits.

Examples

- Example 47
  - What is the number of permutations of 3 objects: a, b, and c?
  - List the permutations.

- Example 49
  - Ten athletes compete in an Olympic event. Gold, silver, and bronze medals are awarded.
  - In how many ways can the awards be made?

Problems

- Problem 6
  - In how many ways can six people be seated in a circle of six chairs? Only relative positions in the circle can be distinguished.
- Problem 10
  - In how many different ways can you seat 11 men and 8 women in a row if the men all sit together and the women all sit together?

Combinations

- What is a combination?
  - A group of r objects chosen from a set of n objects.
  - Order doesn't matter.
- $C(n,r)$ = The number of combinations of r distinct objects chosen from n distinct objects.
- Note $C(n,r) \cdot r! = P(n,r)$
- Formula ($0 \leq r \leq n$)

$$C(n,r) = \frac{P(n,r)}{r!} = \frac{n!}{r!(n-r)!}$$

Examples

- Example 54
  - Ten athletes compete in an Olympic event; three will be declared winners. In how many ways can the winners be chosen?

- Practice 32
  - In how many ways can a committee of 3 be chosen from a group of 12 people?

Example

- Example 55: A committee of 8 students is to be selected from a class consisting of 19 freshman and 34 sophomores.
  a. In how many ways can 3 freshmen and 5 sophomores be selected?
  b. In how many ways can a committee with exactly 1 freshman be selected?
  c. In how many ways can a committee with at most 1 freshman be selected?
  d. In how many ways can a committee with at least 1 freshman be selected?

Problems

- Problem 22
  - Of a company's personnel, 7 work in design, 14 in manufacturing, 4 in testing, 5 in sales, 2 in accounting, and 3 in marketing.
  - A committee of 6 people is to be formed to meet with upper management.
  - In how many ways can the committee be formed
    - if there are no restrictions on membership?
    - if there are exactly 2 members from manufacturing?

Eliminating Duplicates

- Be careful not to count something twice.
- Practice 33
  - A committee of 2 to be chosen from 4 math majors and 3 physics majors must include at least one math major. Compute the following 2 values:
  - $C(4,1) \cdot C(6,1)$
  - $C(7,2) - C(3,2)$
  - Which is the correct answer?

No Repetition

- So far, we've only looked at permutations and combinations without repetitions
  - Example: All strings that can be made using the alphabet $\{p, q, r\}$
  - No repetitions allowed
    - The longest string possible is only 3 characters long
      - *pqr, prq, qpr, qrp, rpq, rqp*

No Repetitions

- Not allowing repetitions results in the restriction $r \leq n$.
- In other words, without repetitions, the most we can choose is n objects out of n objects.
  - If we choose more than n, at least one object will have to be repeated.

Repetitions Allowed

- Allowing repetitions allows r to be greater than n
  - Given our same alphabet $\{p, q, r\}$
  - Choose a string of length 5 built with the alphabet
  - Examples: $pqrpq$, $qqqqq$, $pqrqr$, $prrpq$, etc.
  - How many different strings can be built?
    - $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^5$

Repetitions

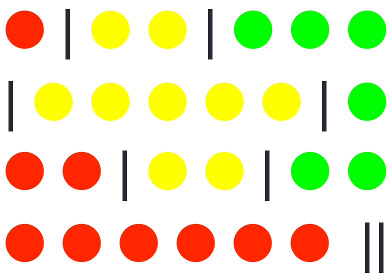
- Permutations
  - Choose r objects (as opposed to r distinct objects) out of n distinct objects
    - n choices for the first object
    - n choices for the second object (because repetitions allowed)
    - n choices for the r^{th} object
    - The total number of permutations is n^r (multiplication principle).

Repetitions Allowed

- Combinations with repetitions allowed
  - Practice 35
    - Six children choose one lollipop each from among a selection of red, yellow, and green lollipops.
      - In how many ways can this be done? (We do not care which child gets which.)
      - What is r? What is n?

Practice 35

- Examples



  - For 3 colors/flavors, how many dividers are there?
- We have 8 slots into which we can insert either a divider or a lollipop.
  - We are choosing which 6 slots should have a lollipop,
  - Or, which 2 slots should have a divider.

Combinations with Repetitions

- The general formula is

$$C(r+n-1,r)=\frac{(r+n-1)!}{r!(n-1)!}$$

Generating Algorithms

- Generating possible permutations and combinations
  - Algorithms exist, some of which use lexicographical ordering, others don't
  - You can examine the algorithms yourself.