CHAPTER 3

Section 3.3

Principle of Inclusion and Exclusion

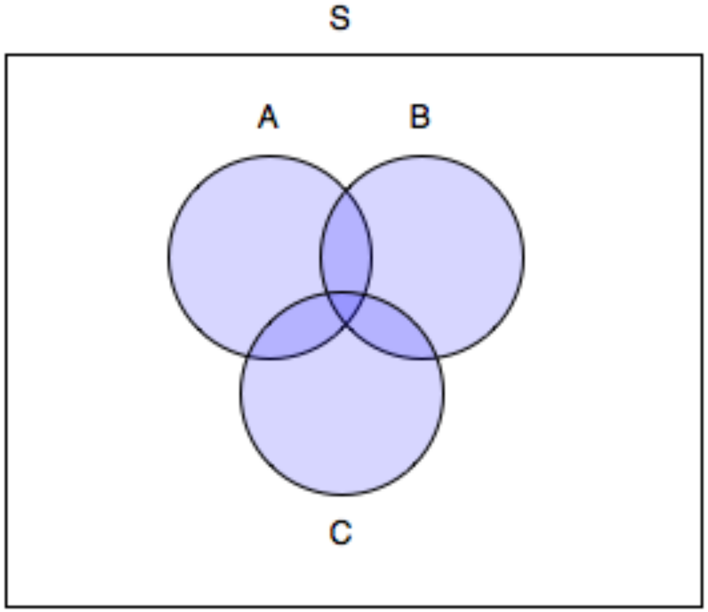
- Principle of Inclusion and Exclusion
  - Another counting principle for solving combinatorics problems
- Given A and B are any subsets of a universal set S
  - $A - B$, $B - A$, and $A \cap B$ are mutually disjoint sets
  - What if you want to count the elements in $A \cup B$?
  - $|A \cup B| = |A| + |B| - |A \cap B|$

Example 40

- A pollster queries 35 voters, all of whom support referendum 1, referendum 2, or both, and finds that 14 voters support referendum 1 and 26 support referendum 2.
  - How many voters support both?
  - A: set of voters that support referendum 1
  - B: set of voters that support referendum 2

Inclusion and Exclusion

- What about 3 sets?



$|A \cup B \cup C| =$

$$|A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Problems

- Problem 1: In a group of 42 tourists, everyone speaks English or French; there are
  - 35 English speakers
  - 18 French speakers
  - How many speak both?

Problems

- Problem 5: In a group of 24 people who like rock, country, and classical music
  - 14 like rock
  - 17 like classical
  - 11 like both rock and country
  - 9 like rock and classical
  - 13 like country and classical
  - 8 like rock, country, and classical
  - How many like country?

Inclusion and Exclusion

- Given the finite sets $A_1, \ldots, A_n$, $n \geq 2$, then

$$
\begin{aligned}
|A_1 \cup \cdots \cup A_n| = &\sum_{1 \leq i \leq n} |A_i| - \sum_{1 \leq i < j \leq n} |A_i \cap A_j| \\
&+ \sum_{1 \leq i < j < k \leq n} |A_i \cap A_j \cap A_k| \\
&- \cdots + (-1)^{n+1} |A_1 \cap \cdots \cap A_n|
\end{aligned}
$$

Pigeonhole Principle

- What happens if more than k pigeons fly into k pigeonholes?
  - One hole will have more than 1 pigeon.
- More generically stated:
  - If more than k items are placed into k bins, then at least one bin contains more than one item.

Problem 17

- How many cards must be drawn from a standard 52-card deck to guarantee 2 cards of the same suit?
  - How many pigeonholes?