

# CHAPTER 3

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## Section 3.2

# Counting

- Combinatorics
  - Branch of math that deals with counting
  - Counting
    - Finding out how many elements are in some finite set
      - Can be harder than it sounds
    - Useful when answering questions about finite resources, measuring efficiency, etc.

# Multiplication Principle

- Tree example using multiplication principle
  - Child is allowed to choose 1 jellybean out of 2 jellybeans (one red and one black) and 1 gummy bear out of 3 gummy bears (yellow, green, and white). How many different **sets** of candy can the child have?

# Multiplication Principle

- Result of the jellybean-gummy bear problem
  - Total number of outcomes for a sequence of events can be determined by multiplying the number of outcomes for the first event by the number of outcomes for the second event.
- Multiplication Principle
  - If there are  $n_1$  possible outcomes for a 1<sup>st</sup> event and  $n_2$  possible outcomes for a 2<sup>nd</sup> event, there are  $n_1 \cdot n_2$  possible outcomes for the sequence of the two events.

# Multiplication Principle

- Useful for counting total number of outcomes for any task that can be broken down into a sequence of successive subtasks.
- Example 25: Last part of your telephone number contains 4 digits. How many such four-digit numbers are there?
  - What are the successive subtasks for this problem?
  - How many four-digit numbers are there if the same digit cannot be used twice?

# Multiplication Principle

- Alabama License Plate (1982)
  - Issued license plates for Madison County, Alabama, conformed to the following format:
    - 47A 1234
    - Where
      - 47 is the county code for Madison County
      - A represents a placeholder for any letter A-Z
      - 1234 each represent a placeholder for a digit from 0-9
  - How many such license plates could be issued?



# Multiplication Principle

- Alabama License Plate (1987)
  - Madison County ran out of license plate numbers for the standard 1982 format:
    - The following overflow format was issued for Madison County:
    - 47 12345
    - Where
      - 47 is the county code for Madison County
      - 12345 each represent a placeholder for a digit from 0-9
  - How many such license plates could be issued?



# Multiplication Principle

- Alabama License Plate Formats for Madison County
  - 1988: Format: 47AB 123
  - 1993: Format 47ABC12
  - 1997: Format 47AB123
    - Ran out in Madison County
      - 2 overflow formats: 47ABC23, then 47A2B34
  - 2002: Format 47A345B
    - Ran out in Madison County
      - Overflow: 47A1B2C
  - 2009: Format 47A34B5
    - Still good so far, possible due to the additional God Bless America plate starting in 2006 and issued concurrently with 2002 and 2009 bases.

# Addition Principle

- What if we want to select a dessert from 3 pies and 4 cakes. In how many ways can this be done?
  - Can the multiplication principle be used here?
- Addition Principle
  - If A and B are disjoint events with  $n_1$  and  $n_2$  possible outcomes, respectively, then the total number of possible outcomes for event “A or B” is  $n_1 + n_2$ .
  - Useful for counting outcomes for tasks that can be broken down into disjoint cases.

# Addition Principle

- Example 30: A customer wants to purchase a vehicle from a dealer. The dealer has 23 cars and 14 trucks in stock.
  - How many selections does the customer have?

# Using Principles Together

- Example 24 again: Child choosing candy
  - We looked at how many different **sets** of candy the child can have after choosing.
  - How many different **ways** can the child choose the candy?
    - Consider choosing jellybeans first and choosing gummy bears first to be 2 disjoint events.
      - Combine the multiplication and addition principles.
      - How many ways when choosing jellybeans first? (Multiplication Principle)
      - How many ways when choosing gummy bears first? (Multiplication Principle)
      - How many total ways? (Addition Principle)

# Using Principles Together

- Alabama License Plate (1982)
  - Consider all Alabama license plates. Format is:
    - 0A 12345 for counties with 1 digit code
      - County codes 1-9
    - 00A 1234 for counties with 2 digit code
      - County codes 10-67
  - How many such license plates could be issued for all of Alabama?



# Problems

- Problem 4
  - A multiple choice exam has 20 question, each with four possible answers, and 10 additional questions, each with 5 possible answers.
  - How many different answer sheets are possible?

# Problems

- Consider the set of binary strings of length 8.
  - Problem 37
    - How many such strings are there?
  - Problem 40
    - How many have 1 as the second digit?
  - Problem 43
    - How many begin with 10 **or** have a 0 as the 3<sup>rd</sup> digit?
  - Problem 46
    - How many contain 2 or more 0s?

# Problems

- Problem 8
  - A, B, C and D are nodes on a computer network. There are 2 paths between A and C, 2 between B and D, 3 between A and B, and 4 between C and D.
  - Along how many routes can a message from A to D be sent?

# Decision Trees

- Example 39: Tony is pitching pennies. Each toss results in heads (H) or tails (T).
  - How many ways can he toss the coin five times without having too heads in a row?
- Practice 25: Draw the decision tree for the number of strings of X' s, Y' s, and Z' s with length 3 that do not have a Z following a Y.