CHAPTER 1

Section 1.3

Representation in Predicate Logic

- Example from book:
  - What if you want to represent the statement:

    For every x, x > 0
  - Can you do it in propositional logic?

  - Predicate logic
    - Uses predicates and quantifiers
    - Above example would be represented as

      $$(\forall x)(x > 0)$$
    - Is this statement true?

Representation in Predicate Logic

- Quantifiers
  - Universal quantifier: $\forall$
    - Means “for all,” “for every”
  - Generically, let $P(x)$ be $x > 0$, then we say

    $$(\forall x)P(x)$$

    where $P(x)$: $x > 0$, domain: all positive integers

  - Example: $P(x)$: *x has a red cover*
    - Domain: all books in the UAHuntsville library
    - What does $(\forall x)P(x)$ mean? Is it true or false?

Representation in Predicate Logic

- Quantifiers
  - Existential quantifier: ∃
    - Means "there exists one," "at least one," "for some"
  - Example

    $(\exists x)P(x)$

    where $P(x)$: $x > 0$, domain: all positive integers
    - There exists an x such that $x > 0$
    - For some x, $x > 0$
    - Is this true?

Practice 15 (page 35)

- What is the truth value of the expression in each of the following interpretations?
  - $P(x)$: x is yellow,
    domain of interpretation: collection of all buttercups
  - $P(x)$: x is yellow,
    domain: collection of all flowers
  - $P(x)$: x is a plant,
    domain: all flowers
  - $P(x)$: x is either positive or negative,
    domain: the integers

Practice 16 (page 36)

- Construct an interpretation in which $(\forall x)P(x)$ has the value true.
  - Interpretation: defined on page 37
- Construct an interpretation in which $(\forall x)P(x)$ has the value false.
- Can you find one interpretation in which both $(\forall x)P(x)$ is true and $(\exists x)P(x)$ is false?
- Can you find one interpretation in which both $(\forall x)P(x)$ is false and $(\exists x)P(x)$ is true?

Translating English Statements

- Translating English statements into predicate logic statements
  - All cows eat grass.
    - Domain: whole world
  - Predicates
    - $C(x)$: x is a cow
    - $G(x)$: x eats grass
  - How do we represent the statement in predicate logic?
    - Reword: For any thing, if that thing is a cow, it eats grass.

Translating English Statements

- Translating English statements into predicate logic statements
  - Some cow eats grass.
    - Domain: whole world
  - Predicates
    - $C(x)$: x is a cow
    - $G(x)$: x eats grass
  - How do we represent the statement in predicate logic?
    - Reword: There is at least one thing that is a cow and it eats grass.

n-ary Predicates

- So far we have see unary predicates.
- Can also have binary (and ternary, etc.) predicates.
  - Example: $(\forall x)(\exists y)Q(x,y)$
  - For every x, there exists a y such that $Q(x, y)$.
- Example 20
  - $Q(x, y)$: $x < y$, domain is the set of integers
  - What does $(\forall x)(\exists y)Q(x,y)$ mean in this context?
  - What about $(\exists y)(\forall x)Q(x,y)$

Practice 18 (page 40)

- Use the following predicate symbols to construct wffs that express the given statements.
  - Predicates
    - $S(x)$: x is a student
    - $I(x)$: x is intelligent
    - $M(x)$: x likes music
    - Domain: the collection of all people
  - Statements
    - All students are intelligent.
    - Some intelligent students like music.
    - Everyone who likes music is a stupid student.
    - Only intelligent students like music.

Practice 19 (page 40)

- Translate English into symbolic form
  - Predicates
    - $F(x)$: x is a fruit
    - $V(x)$: x is a vegetable
    - $S(x,y)$: x is sweeter than y
    - Domain: whole world
  - Statements
    - Some vegetable is sweeter than all fruits.
    - Every fruit is sweeter than all vegetables.
    - Every fruit is sweet than some vegetable.
    - Only fruits are sweeter than vegetables.

Negation

- Given
  - $B(x)$: x is beautiful
- Negate the statement $(\forall x)B(x)$
  - Negation:

$$[(\forall x)B(x)]' = ?$$

- Negate the statement $(\exists x)B(x)$
  - Negation:

$$[(\exists x)B(x)]' = ?$$

§ 1.3, Problem #20 (page 48)

- Three forms of negation are given for each statement. Which is correct?
  - Nobody is perfect.
    - Everyone is imperfect.
    - Everyone is perfect.
    - Someone is perfect.
  - All swimmers are tall.
    - Some swimmer is not tall.
    - There are no tall swimmers.
    - Every swimmer is short.
  - Every planet is cold and lifeless.
    - No planet is cold and lifeless.
    - Some planet is not cold and not lifeless.
    - Some planet is not cold or not lifeless.

Validity

- Analogous to a tautology of propositional logic
- Truth of a predicate wff depends on the interpretation.
- A predicate wff is valid if it is true in all possible interpretations just like a propositional wff is a tautology if it is true for all rows of the truth table.

Validity Examples

$$(\forall x)P(x) \rightarrow (\exists x)P(x)$$

- Valid - if every object of the domain has a certain property, then there exists an object of the domain that has the same property.

$$(\forall x)P(x) \rightarrow P(a)$$

- Valid – a is a member of the domain of x.

$$(\exists x)P(x) \rightarrow (\forall x)P(x)$$

- Not valid – the property cannot be valid for all objects in the domain just because it is valid for some objects of the domain.
- Counterexample: Say $P(x) = x \textit{ is even}$, then there exists an integer that is even but not every integer is even.

$$(\forall x)[P(x) \vee Q(x)] \rightarrow (\forall x)P(x) \vee (\forall x)Q(x)$$

- Invalid - Counterexample: Say $P(x) = x \textit{ is even}$ and $Q(x) = x \textit{ is odd}$
- The hypothesis is true but not the conclusion, because it is not the case that every integer is even or that every integer is odd.

§ 1.3, Problem #10

- Given
  - $B(x)$: x is a ball
  - $R(x)$: x is round
  - $S(x)$: x is a soccer ball
- Write each statements as a predicate wff
  - All balls are round.
  - Not all balls are soccer balls.
  - All soccer balls are round.
  - Some balls are not round.
  - Some balls are round but soccer balls are not.
  - Every round ball is a soccer ball.
  - Only soccer balls are round balls.
  - If soccer balls are round, then all balls are round.