CHAPTER 1

Section 2

Propositional Logic

- Reasoning in formal logic
  - Determining the truth of an argument
- Argument
  - A sequence of statements in which the conjunction of the initial statements (the premises/hypotheses) is said to imply the final statement (the conclusion).

- Statements can also be called propositions

$$P_1 \wedge P_2 \wedge \ldots \wedge P_n \rightarrow Q$$

Validity of Arguments

- Valid Argument
  - The propositional wff

$$P_1 \wedge P_2 \wedge \ldots \wedge P_n \rightarrow Q$$

    is a valid argument when it is a tautology.

- Informally, an argument is valid whenever the truth of the hypotheses leads to the conclusion.

- We are **not** proving the conclusion is true
  - We are proving the conclusion logically follows from the hypotheses

Validity of Arguments

- Example:
  - P_1: Neil Armstrong was the first human to step on the moon.
  - P_2: Mars is a red planet.
  - Q: No human has ever been to Mars.
- Is this a valid argument?

Validity of Arguments

- How to arrive at a valid argument?
  - Truth table
  - Tautology test
  - Proof sequence
- Proof Sequence
  - Sequence of wffs in which each wff is either a hypothesis or the result of applying one of the formal system's derivation rules to earlier wffs in the sequence.

Rules for Propositional Logic

- Derivation Rules
  - Equivalence Rules
    - State that certain pairs of wffs are equivalent
      - One can be substituted for the other with no change to truth values
      - Work in both directions
    - Allows individual wffs to be rewritten
  - Inference Rules
    - Allows new wffs to be derived
    - Work in only one direction

Equivalence Rules

- Table 1.12, page 24

Expression	Equivalent to	Rule - Abbreviation
$R \vee S$ $R \wedge S$	$S \vee R$ $S \wedge R$	Commutative – comm
$(R \vee S) \vee Q$ $(R \wedge S) \wedge Q$	$R \vee (S \vee Q)$ $R \wedge (S \wedge Q)$	Associative – assoc
$(R \vee S)'$ $(R \wedge S)'$	$R' \wedge S'$ $R' \vee S'$	De Morgan's laws – De Morgan
$R \rightarrow S$	$R' \vee S$	Implication – imp
R	$(R')'$	Double Negation – dn
$R \leftrightarrow Q$	$(R \rightarrow Q) \wedge (Q \rightarrow R)$	Equivalence – equ

Proof Sequence Example

- Use equivalence rules to prove the argument is valid

$$((A' \vee B') \vee C) \rightarrow ((A \wedge B) \rightarrow C)$$

- Hypothesis

$$(A' \vee B') \vee C$$

- Conclusion

$$(A \wedge B) \rightarrow C$$

Inference Rules

- Table 1.13, page 25

From	Can Derive	Rule - Abbreviation
P $P \rightarrow Q$	Q	Modus ponens – mp
$P \rightarrow Q$ Q'	P'	Modus tollens – mt
P Q	$P \wedge Q$	Conjunction – con
$P \wedge Q$	P Q	Simplification – sim
P	$P \vee Q$	Addition – add

Example: §1.2, Problem10

- Justify each step in the proof sequence of

  $$[A \rightarrow (B \lor C)] \land B' \land C' \rightarrow A'$$

- Proof sequence

  1. $A \rightarrow (B \lor C)$

  2. B'

  3. C'

  4. $B' \land C'$

  5. $(B \lor C)'$

  6. A'

Example: §1.2, Problem 15

- Use propositional logic to prove the argument is valid

$$A' \wedge (A \vee B) \rightarrow B$$

- First, must identify hypotheses and conclusion.
  - Hypotheses

    $$A'$$

    $$A \vee B$$
  - Conclusion

    $$B$$

Example: §1.2, Problem 17

- Prove the following argument is valid

$$(A' \rightarrow B') \wedge B \wedge (A \rightarrow C) \rightarrow C$$

- First, must identify hypotheses and conclusion.

Deduction Method

- Can use to prove an argument of the form

$$P_1 \wedge P_2 \wedge \ldots \wedge P_n \rightarrow (R \rightarrow Q)$$

- Deduction method allows for the use of R as an additional hypothesis and thus prove

$$P_1 \wedge P_2 \wedge \ldots \wedge P_n \wedge R \rightarrow Q$$

- Example: Prove the following argument is valid

$$(A \rightarrow B) \wedge (B \rightarrow C) \rightarrow (A \rightarrow C)$$

- This rule is called hypothetical syllogism
- Many such other rules can be derived from existing rules which thus provide easier and faster proofs.

Example: §1.2, Problem 22

- Prove the following argument is valid

$$[A \rightarrow (B \vee C)] \wedge C' \rightarrow (A \rightarrow B)$$

- First, must identify hypotheses and conclusion.
  - Use deduction method

More Inference/Equivalence Rules

- Table 1.14, page 33

From	Can Derive	Rule - Abbreviation
$P \rightarrow Q, Q \rightarrow R$	$P \rightarrow R$	Hypothetical Syllogism – hs
$P \vee Q, P'$	Q	Disjunctive Syllogism – ds
$P \rightarrow Q$	$Q' \rightarrow P'$	Contraposition - cont
P	$P \wedge P$	Self-reference – self
$P \vee P$	P	Self-reference – self
$(P \wedge Q) \rightarrow R$	$P \rightarrow (Q \rightarrow R)$	Exportation – exp
P, P'	Q	Inconsistency – inc
$P \wedge (Q \vee R)$	$(P \wedge Q) \vee (P \wedge R)$	Distributive – dist
$P \vee (Q \wedge R)$	$(P \vee Q) \wedge (P \vee R)$	Distributive - dist

Example: §1.2, Problem 36

- Prove the following argument is valid

$$(A \lor B) \land (A \rightarrow C) \land (B \rightarrow C) \rightarrow C$$

- First, must identify hypotheses and conclusion.

Example 14 (page 26)

- Prove the following argument is valid

$$A \wedge (B \rightarrow C) \wedge [(A \wedge B) \rightarrow (D \vee C')] \wedge B \rightarrow D$$

- First, must identify hypotheses and conclusion.

Verbal Argument Proof

- §1.2, Problem 43
  - The crop is good, but there is not enough water. If there is a lot of rain or not a lot of sun, then there is enough water. Therefore, the crop is good and there is a lot of sun.
  - C: the crop is good
  - W: there is enough water
  - R: there is a lot of rain
  - S: there is a lot of sun
- Prove the argument is valid.

Argument from page 1

- Prove the following argument is valid
  - If my client is guilty, then the knife was in the drawer. Either the knife was not in the drawer or Jason Pritchard saw the knife. If the knife was not there on Oct. 10, it follows that Jason Pritchard did not see the knife. Furthermore, if the knife was there on Oct. 10, then the knife was in the drawer and also the hammer was in the barn. But we all know that the hammer was not in the barn. Therefore, ladies and gentlemen of the jury, my client is innocent.

  - G: client is guilty   O: knife there Oct. 10
  - K: knife was in the drawer   H: hammer in the barn
  - J: Jason Pritchard saw knife