CHAPTER 1
SECTION 1 & 2

8/28/2012

Tautology

- Tautology: A wff whose truth value is always true
- Example: A ∨ A' is always true

A	A'	A ∨ A'
0	1	**1**
1	0	**1**

- Today is Tuesday or today is not Tuesday

Contradiction

- Contradiction: A wff whose truth value is always false
- Example: $A \wedge A'$ is always false

A	A'	$A \wedge A'$
0	1	**0**
1	0	**0**

- Today is Tuesday and today is not Tuesday

Equivalent Well-Formed Formulas

- Given 2 wffs P & Q
  - If $P \leftrightarrow Q$ is a tautology (P and Q's values always match) then, P and Q are equivalent wffs.

$$P \Leftrightarrow Q$$

  - If P & Q are equivalent wffs then, P can be replaced by Q in an wff R containing P, resulting in a wff R_Q that is equivalent to R
    - So, you can replace P with Q and everything (truth value of entire wff) is the same
    - Useful in performing proofs

Equivalent Well-Formed Formulas

- Example 6, page 9
  - Let $R:(A \to B) \to B$

    $P:(A \to B)$
  - From 7d, P is equivalent to $B' \to A'$
  - Replace P with Q in R to get

    $R_Q:(B' \to A') \to B$
  - Use truth table to show $R \Leftrightarrow R_Q$

Some Tautological Equivalences

- Bottom of page 8

$1a.\ A \vee B \Leftrightarrow B \vee A$	$1b.\ A \wedge B \Leftrightarrow B \wedge A$	Commutative Properties
$2a.\ (A \vee B) \vee C \Leftrightarrow A \vee (B \vee C)$	$2b.\ (A \wedge B) \wedge C \Leftrightarrow A \wedge (B \wedge C)$	Associative Properties
$3a.\ A \vee (B \wedge C) \Leftrightarrow (A \vee B) \wedge (A \vee C)$	$3b.\ A \wedge (B \vee C) \Leftrightarrow (A \wedge B) \vee (A \wedge C)$	Distributive Properties
$4a.\ A \vee 0 \Leftrightarrow A$	$4a.\ A \wedge 1 \Leftrightarrow A$	Identity Properties
$5a.\ A \vee A' \Leftrightarrow 1$	$5a.\ A \wedge A' \Leftrightarrow 0$	Complement Properties

Dual of Equivalence

- The dual of an equivalence is obtained by:
  - Replacing $\vee$ with $\wedge$
  - Replacing $\wedge$ with $\vee$
  - Replacing 0 with 1, and
  - Replacing 1 with 0
- Tautological Equivalences on page 8
  - Second part of each set of equivalences is dual of first

De Morgan's Law

- Very useful equivalence along with its dual is De Morgan's Law

$$(A \vee B)' \Leftrightarrow A' \wedge B'$$

$$(A \wedge B)' \Leftrightarrow A' \vee B'$$

De Morgan's Law

- Example 7, page 10
  - Express the logical expression from the computer program in symbolic form
  - Use tautological equivalences to simplify the expression

1.2 Propositional Logic

- Reasoning in formal logic
  - Determining the truth of an argument
- Argument
  - A sequence of statements in which the conjunction of the initial statements (the premises/hypotheses) is said to imply the final statement (the conclusion).

$$P_1 \wedge P_2 \wedge \ldots \wedge P_n \rightarrow Q$$

- Statements can also be called propositions

Validity of Arguments

- Valid Argument
  - The propositional wff

$$P_1 \wedge P_2 \wedge \ldots \wedge P_n \rightarrow Q$$

  is a valid argument when it is a tautology.

- Informally, an argument is valid whenever the truth of the hypotheses leads to the conclusion.
- We are **not** proving the conclusion is true
  - We are proving the conclusion logically follows from the hypotheses

Validity of Arguments

- Example:
  - P_1: Neil Armstrong was the first human to step on the moon.
  - P_2: Mars is a red planet.
  - Q: No human has ever been to Mars.
- Is this a valid argument?

Validity of Arguments

- How to arrive at a valid argument?
  - Truth table
  - Tautology test
  - Proof sequence ←
- Proof Sequence
  - Sequence of wffs in which each wff is either a hypothesis or the result of applying one of the formal system's derivation rules to earlier wffs in the sequence.

Rules for Propositional Logic

- Derivation Rules
  - Equivalence Rules
    - State that certain pairs of wffs are equivalent
      - One can be substituted for the other with no change to truth values
      - Work in both directions
    - Allows individual wffs to be rewritten
  - Inference Rules
    - Allows new wffs to be derived
    - Work in only one direction

Equivalence Rules

- Table 1.12, page 24

Expression	Equivalent to	Rule - Abbreviation
$R \vee S$ $R \wedge S$	$S \vee R$ $S \wedge R$	Commutative – comm
$(R \vee S) \vee Q$ $(R \wedge S) \wedge Q$	$R \vee (S \vee Q)$ $R \wedge (S \wedge Q)$	Associative – assoc
$(R \vee S)'$ $(R \wedge S)'$	$R' \wedge S'$ $R' \vee S'$	De Morgan's laws – De Morgan
$R \rightarrow S$	$R' \vee S$	Implication – imp
R	$(R')'$	Double Negation – dn
$R \leftrightarrow Q$	$(R \rightarrow Q) \wedge (Q \rightarrow R)$	Equivalence – equ

Proof Sequence Example

- Use equivalence rules to prove the argument is valid

  $((A' \vee B') \vee C) \rightarrow ((A \wedge B) \rightarrow C)$

- Hypothesis

  $(A' \vee B') \vee C$

- Conclusion

  $(A \wedge B) \rightarrow C$

Inference Rules

- Table 1.13, page 25

From	Can Derive	Rule - Abbreviation
P $P \to Q$	Q	Modus ponens – mp
$P \to Q$ Q'	P'	Modus tollens – mt
P Q	$P \wedge Q$	Conjunction – con
$P \wedge Q$	P Q	Simplification – sim
P	$P \vee Q$	Addition – add

Example: §1.2, Problem 12

- Use propositional logic to prove the argument is valid

  $$A' \wedge (B \to A) \to B'$$

- First, must identify hypotheses and conclusion.
  - Hypotheses

    $$A'$$

    $$B \to A$$

  - Conclusion

    $$B'$$

Example: §1.2, Problem10

- Justify each step in the proof sequence of

  $$[A \to (B \vee C)] \wedge B' \wedge C' \to A'$$

- Proof sequence

  1. $A \to (B \vee C)$
  2. B'
  3. C'
  4. $B' \wedge C'$
  5. $(B \vee C)'$
  6. A'

Deduction Method

- Can use to prove an argument of the form

$$P_1 \wedge P_2 \wedge \ldots \wedge P_n \rightarrow (R \rightarrow Q)$$

- Deduction method allows for the use of R as an additional hypothesis and thus prove

$$P_1 \wedge P_2 \wedge \ldots \wedge P_n \wedge R \rightarrow Q$$

- Example: Prove the following argument is valid

$$(A \rightarrow B) \wedge (B \rightarrow C) \rightarrow (A \rightarrow C)$$

  - This rule is called hypothetical syllogism
  - Many such other rules can be derived from existing rules which thus provide easier and faster proofs.

Example

- Prove the argument is valid

$$A \wedge (B \rightarrow C) \wedge [(A \wedge B) \rightarrow (D \vee C')] \wedge B \rightarrow D$$

  - First, identify hypotheses and conclusion
  - Then, use the inference and equivalence rules to reach conclusion

Example: §1.2, Problem 13

- Prove the following argument is valid

  $$(A \to B) \wedge [A \to (B \to C)] \to (A \to C)$$

- First, must identify hypotheses and conclusion.
  - Use deduction method

More Inference/Equivalence Rules

- Table 1.14, page 33

From	Can Derive	Rule - Abbreviation
$P \to Q, Q \to R$	$P \to R$	Hypothetical Syllogism – hs
$P \vee Q, P'$	Q	Disjunctive Syllogism – ds
$P \to Q$	$Q' \to P'$	Contraposition - cont
P	$P \wedge P$	Self-reference – self
$P \vee P$	P	Self-reference – self
$(P \wedge Q) \to R$	$P \to (Q \to R)$	Exportation – exp
P, P'	Q	Inconsistency – inc
$P \wedge (Q \vee R)$	$(P \wedge Q) \vee (P \wedge R)$	Distributive – dist
$P \vee (Q \wedge R)$	$(P \vee Q) \wedge (P \vee R)$	Distributive - dist

Example: §1.2, Problem 16

- Prove the following argument is valid

$$[A \to (B \to C)] \wedge (A \vee D') \wedge B \to (D \to C)$$

- First, must identify hypotheses and conclusion.

Example 14 (page 26)

- Prove the following argument is valid

$$A \wedge (B \to C) \wedge [(A \wedge B) \to (D \vee C')] \wedge B \to D$$

- First, must identify hypotheses and conclusion.

Verbal Argument Proof

- §1.2, Problem 43
  - The crop is good, but there is not enough water. If there is a lot of rain or not a lot of sun, then there is enough water. Therefore, the crop is good and there is a lot of sun.
  - C: the crop is good
  - W: there is enough water
  - R: there is a lot of rain
  - S: there is a lot of sun
- Prove the argument is valid.

Argument from page 1

- Prove the following argument is valid
  - If my client is guilty, then the knife was in the drawer. Either the knife was not in the drawer or Jason Pritchard saw the knife. If the knife was not there on Oct. 10, it follows that Jason Pritchard did not see the knife. Furthermore, if the knife was there on Oct. 10, then the knife was in the drawer and also the hammer was in the barn. But we all know that the hammer was not in the barn. Therefore, ladies and gentlemen of the jury, my client is innocent.

  - G: client is guilty
  - K: knife was in the drawer
  - J: Jason Pritchard saw knife

  O: knife there Oct. 10
  H: hammer in the barn