CHAPTER 1

Section 1

Formal Logic

- Consists of
  - Representation
  - Reasoning

- Example argument from book (page 1)

  If my client is guilty, then the knife was in the drawer. Either the knife was not in the drawer or Jason Pritchard saw the knife. If the knife was not there on Oct. 10, it follows that Jason Pritchard did not see the knife. Furthermore, if the knife was there on Oct. 10, then the knife was in the drawer and also the hammer was in the barn. But we all know that the hammer was not in the barn. Therefore, ladies and gentlemen of the jury, my client is innocent.

Formal Logic: Representation

- Formal logic can be used to represent the statements we use in English to communicate facts or information
- **Statement**
  - A sentence that is either true or false (T or F, 1 or 0)
  - If the item cannot be either true or false (e.g., a question), then it is not a statement.
  - Also called a proposition
- Example 1 (page 2)

Formal Logic: Representation

- **Connectives**
  - Connect different statements to form compound statements
    - Conjunction
    - Disjunction
    - Implication
    - Equivalence
    - Negation

Logical Connectives

- **Conjunction**: AND
  - $A \wedge B$ means "A and B"
    - A, B are called the **conjuncts** of the expression
  - $A \wedge B$ is true when both A and B are true.
    - If the compound statement $A \wedge B$ is true, what does that tell us about the statement A and the statement B individually?
    - If the statement A is true and the statement B is true, what do we know about the compound statement $A \wedge B$?
  - Truth Table

A	B	$A \wedge B$
0	0	
0	1	
1	0	
1	1	

Logical Connectives

- **Disjunction**: OR
  - $A \vee B$ means "A or B"
    - A, B are called the **disjuncts** of the expression
  - $A \vee B$ is true when either A is true or B is true (or both).
    - If the compound statement $A \vee B$ is true, what does that tell us about the statement A and the statement B individually?
    - If the statement B is true, what do we know about the compound statement $A \vee B$?
  - Truth Table

A	B	$A \vee B$
0	0	
0	1	
1	0	
1	1	

Logical Connectives

- **Implication**: IMPLIES; If, then
  - A → B means "A implies B" or "If A, then B"
    - A is the antecedent, B is the consequent
    - If A is true, then B must be true.
  - A → B is true when either A is false or B is true.
    - If A is false, what does that tell us about the statement A → B?
    - If A is true, what does that tell us about the statement A → B?
    - If B is true, what does that tell us about the statement A→ B?
    - Is B is false, what does that tell us about the statement A → B?
  - Truth Table

A	B	A → B
0	0	
0	1	
1	0	
1	1	

Logical Connectives

- **Equivalence**: EQUIVALENT
  - A ↔ B means "A is equivalent to B"
    - Shorthand for (A → B) ∧ (B → A)
  - A ↔ B is true only when the truth values of A and B match.

  - Truth Table

A	B	A → B	B → A	(A → B) ∧ (B → A)	A ↔ B
0	0				
0	1				
1	0				
1	1				

Logical Connectives

- **Negation**: NOT
  - A′ means “Not A”
    - A′ is a unary connective, rather than a binary connective
  - A′ is true when A is false.
    - If A is true, what do we know about A′?
    - If A′ is false, what do we know about A?
  - Truth Table

A	A′
0	
1	

Well-Formed Formulas

- New expressions can be formed by combining statement letters, connectives, and parentheses.

$$((A \wedge B) \rightarrow (C \rightarrow (D \vee E))) \leftrightarrow F$$

- There are syntax rules.
- The following is not a legitimate string – it is not a valid statement.

$$(A \rightarrow\rightarrow (C((D)$$

- An expression that is a legitimate string is called a **well-formed formula** (**wff**).

Precedence Rules

- Define the order in which connectives are applied

- Order of precedence
  1. connectives within parentheses, innermost parentheses first
  2. $'$
  3. $\wedge$, $\vee$
  4. $\rightarrow$
  5. $\leftrightarrow$

- Main connective
  - The connective to be applied last

Tautology

- Tautology: A wff whose truth value is always true
- Example: A ∨ A′ is always true

A	A′	A ∨ A′
0	1	**1**
1	0	**1**

- Today is Tuesday or today is not Tuesday

Contradiction

- Contradiction: A wff whose truth value is always false
- Example: $A \wedge A'$ is always false

A	A′	A ∧ A′
0	1	**0**
1	0	**0**

- Today is Tuesday and today is not Tuesday

Equivalent Well-Formed Formulas

- Given 2 wffs P and Q
  - If $P \leftrightarrow Q$ is a tautology (P and Q's values always match) then, P and Q are equivalent wffs.

$$P \Leftrightarrow Q$$

  - If P and Q are equivalent wffs then, P can be replaced by Q in a wff R containing P, resulting in a wff R_Q that is equivalent to R
    - So, you can replace P with Q and everything (truth value of entire wff) is the same
    - Useful in performing proofs

Equivalent Well-Formed Formulas

- Example 6, page 9
  - Let

$$R : (A \to B) \to B$$

$$P : A \to B$$

  - If we know that P is equivalent to $B' \to A'$
  - Replace P with Q in R to get

$$R_Q : (B' \to A') \to B$$

  - Use truth table to show $R \Leftrightarrow R_Q$

Some Tautological Equivalences

- Bottom of page 8

$1a.\ A \vee B \Leftrightarrow B \vee A$	$1b.\ A \wedge B \Leftrightarrow B \wedge A$	Commutative Properties
$2a.\ (A \vee B) \vee C \Leftrightarrow A \vee (B \vee C)$	$2b.\ (A \wedge B) \wedge C \Leftrightarrow A \wedge (B \wedge C)$	Associative Properties
$3a.\ A \vee (B \wedge C) \Leftrightarrow (A \vee B) \wedge (A \vee C)$	$3b.\ A \wedge (B \vee C) \Leftrightarrow (A \wedge B) \vee (A \wedge C)$	Distributive Properties
$4a.\ A \vee 0 \Leftrightarrow A$	$4a.\ A \wedge 1 \Leftrightarrow A$	Identity Properties
$5a.\ A \vee A' \Leftrightarrow 1$	$5a.\ A \wedge A' \Leftrightarrow 0$	Complement Properties

Dual of Equivalence

- The dual of an equivalence is obtained by:
  - Replacing $\vee$ with $\wedge$
  - Replacing $\wedge$ with $\vee$
  - Replacing 0 with 1, and
  - Replacing 1 with 0
- Tautological Equivalences on page 8
  - Second part of each set of equivalences is dual of first

De Morgan's Law

- Very useful equivalence along with its dual is De Morgan's Law

$$(A \vee B)' \Leftrightarrow A' \wedge B'$$

$$(A \wedge B)' \Leftrightarrow A' \vee B'$$

De Morgan’s Law

- Example 7, page 10
  - Express the logical expression from the computer program in symbolic form
  - Use tautological equivalences to simplify the expression