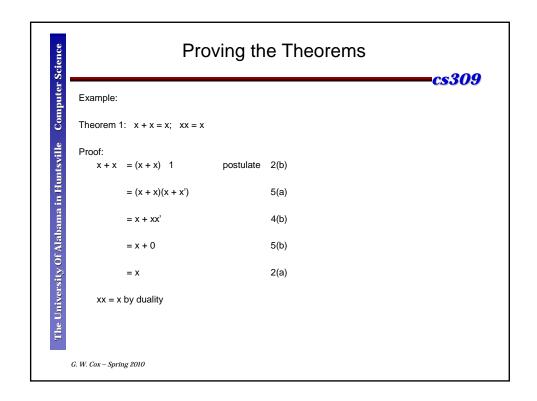
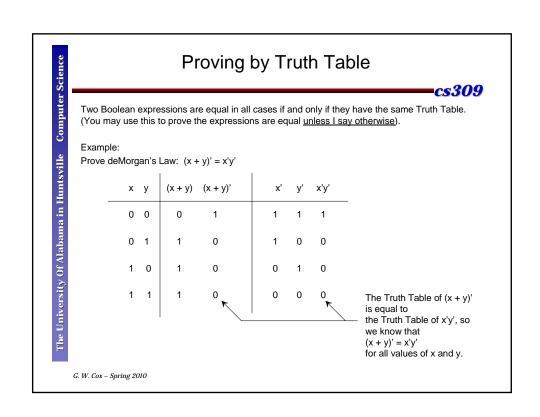
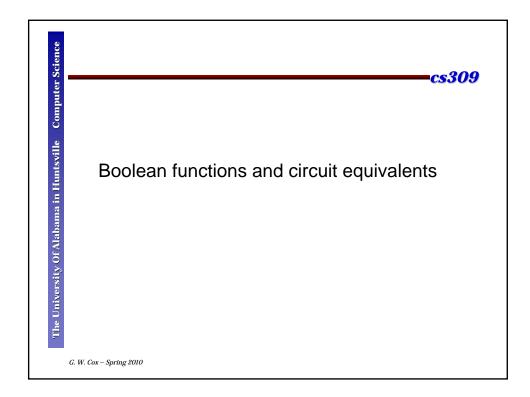
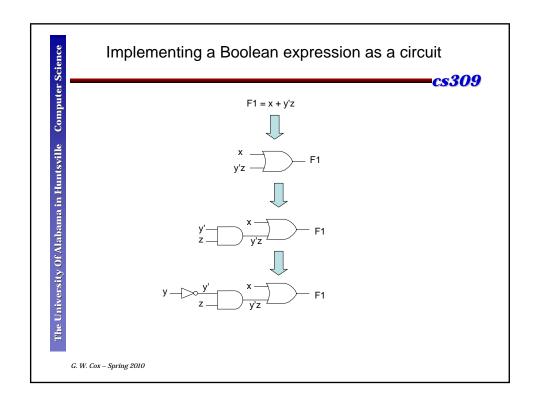


	(a)	(b)
Postulate 2	x + 0 = x	x •1 = x
Postulate 5	x + x' = 1	xx' = 0
Theorem 1	X + X = X	xx = x
Theorem 2	x + 1 = 1	x • 0 = 0
Theorem 3 (involution)	(x')' = x	
Postulate 3 (commutative)	x + y = y + x	xy = yx
Theorem 4 (associative)	x + (y + z) = (x + y) = z	x(yz) = (xy)z
Postulate 4 (distributive)	x(y+z) = xy + xz	x + yz = (x + y)(x + z)
Theorem 5 (deMorgan's Law)	(x + y)' = x'y'	(xy)' = x' + y'
Theorem 6 (absorption)	x + xy = x	x(x+y)=x









## Simplifying expressions

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- There are many different ways to write the same expression
  - Example: x'y'z' + x'yz + xy' = xy' + x'z
- · Different forms of the expression will require different numbers of gates to implement
  - Proof? See page 45 in text
- Generally, longer expressions with more terms require more gates and/or more complex gates
  - More gates → higher power, higher cost, larger size, ...
- So finding a way to simplify expressions will pay off in terms of the circuits we design

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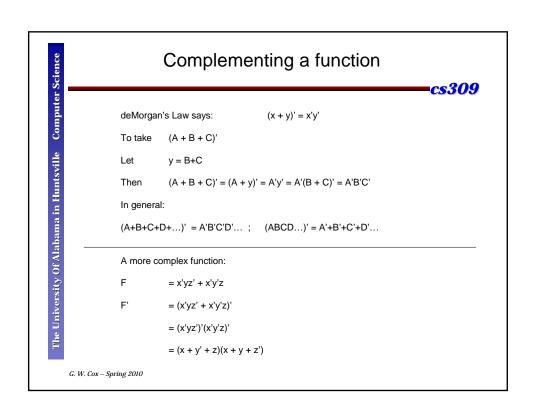
### A metric for use in simplifying expressions

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- Define a "literal" as each occurrence of a variable in the expression
  - Example: F2 = x'y'z + x'yz + xy' → 8 literals
- If we can write the expression with fewer literal, we will consider it to be simpler (and to take fewer gates).
- Note that this is a rule of thumb and does not always give an optimum answer

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```
Simplifying expressions using the postulates and
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                                    theorems of Boolean Algebra
                                                                                                                      cs309
            From page 46-47 of text
             1. x(x' + y)
                                                                                                      (3 literals)
                                          = xx' + xy
                                                                                      p5b
p2a
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                                          = 0 + xy
                                                                                                      (2 literals)
                                          = xy
                                                                                       The dual of (1)
                 x + x'y
            3. (x + y)(x + y')
                                                                                                      (4 literals)
                                                                                       p4b
                                                                                      p5b
p2a
                                                                                                      (1 literal)
                                          = X
                xy + x'z + yz
                                                                                                      (6 literals)
                                                                                      p2b
p5a
p4a
                                          = xy + x'z + yz(1)
                                          = xy + x'z + yz(x + x')
= xy + x'z + yzx + yzx'
                                                                                                                        The
                                                                                       p3b twice
                                          = xy + x'z + xyz + x'zy
                                          = xy + xyz + x'z + x'zy 
= xy1 + xyz + x'z1 + x'zy 
= xy(1 + z) + x'z(1 + y)
                                                                                                                        Consensus
                                                                                       p3a twice
                                                                                       p2b twice
                                                                                                                        Theorem
                                                                                       p4a twice
                                          = xy(1+2)+1
= xy1 + x'z1
= xy + x'z
                                                                                       T2a twice
                                                                                                     (4 literals)
                                                                                       p2a twice
                                                                                       The dual of (4)
            5. (x+y)(x'+z)(y+z)
                                          = (x+y)(x'+z)
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```



# A shortcut for complementing a function

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To complement a function, you can take the dual of the function, and complement each literal.

For the previous example:

$$\mathsf{F} \qquad = \mathsf{x}'\mathsf{y}\mathsf{z}' + \mathsf{x}'\mathsf{y}'\mathsf{z}$$

dual of 
$$F = (x' + y + z')(x' + y' + z)$$

so F' = 
$$(x + y' + z)(x + y + z')$$

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Standard forms of Boolean Expressions

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### **Definitions** cs309 Product term a term consisting of literals ANDed together » Example: AB'F The University Of Alabama in Huntsville a Product term in which all variables appear Minterm » Example: ABC'D where A,B,C, and D are the variables of the function Sum term a term consisting of literals ORed together » Example: A + B' + F a Sum term in which all variables appear Maxterm » Example: A + B + C + D where A, B, C, and D are the variables of the function G. W. Cox - Spring 2010

# SOP and Canonical SOP Form A function is in Sum of Products (SOP) form if it is written as product terms ORed together Example: f(x y z) = xy'z + xz + y A function is in Canonical SOP form if it is in SOP form and all terms are minterms Example: g(x y z) = xy'z + x'yz + xyz G. W. Cax - Spring 2010

### POS and Canonical POS form

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- A function is in Product of Sums (POS) form if it is written as sum terms ANDed together
  - Example: f(x y z) = (x + y + z) (x + z') (y)
- A function is in Canonical POS form if it is written in POS form and all terms are Maxterms
  - Example: g(x y z) = (x' + y' + z') (x + y + z)

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### Minterms and the Truth Table cs309 Each row of a Truth Table corresponds to a minterm x y z | f(x y z)minterm 0 0 0 m<sub>0</sub> x' y' z' 0 0 1 1 m<sub>1</sub> x'y'z The 1's of the Truth Table m<sub>2</sub> x'y z' 0 1 0 0 show the minterms that are in the Canonical SOP expression 0 1 1 m<sub>3</sub> x'y z 1 0 0 1 0 1 m<sub>5</sub> x y'z 1 1 0 0 m<sub>6</sub> xyz' 1 1 1 m<sub>7</sub> xyz $f(x\ y\ z) = x'y'z + xy'z' + xy'z + x\ y\ z$ Minterm List Form: $f(x \ y \ z) = \Sigma m(1, 4, 5, 7)$ G. W. Cox - Spring 2010

