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# Chapter 1

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# Digital Logic

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- Electronic devices used to build computer circuits
  - Can take on Two States represented by voltage levels (high and low).
  - We can think of these as representing any 2-valued system:
    - “1” or “0” (“binary digits” or bits)
    - “True” or “False”
    - “on” or “off”
    - “live” or “dead”
  - We can build on these 2-states to implement:
    - Number representations and arithmetic operations (binary numbers)
    - Codes for various purposes (character codes, error codes, addresses…)
    - Logical operations

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# Grouping bits



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- We can group bits to represent larger numbers of things:

| | | | | | |
|---|---|---|---|---|---|
| 0 | Black | 00 | Black | 000 | Black |
| 1 | White | 01 | White | 001 | White |
| | | 10 | Blue | 010 | Blue |
| | | 11 | Red | 011 | Red |
| | | | | 100 | Yellow |
| | | | | 101 | Orange |
| | | | | 110 | Pink |
| | | | | 111 | Green |

*With n bits, we can represent $2^n$ values or give a unique identifier to $2^n$ things*

- Common terms of reference:

bit: 1 binary digit
byte: 8 bits
word: often 16 bits (may vary)



# Counting in Binary



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| Decimal | Binary |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 10 |
| 3 | 11 |
| 4 | 100 |
| 5 | 101 |
| 6 | 110 |
| 7 | 111 |
| 8 | 1000 |
| 9 | 1001 |
| 10 | 1010 |
| 11 | 1011 |
| 12 | 1100 |
| 13 | 1101 |
| 14 | 1110 |
| 15 | 1111 |

*Note that with n bits,we can give a unique number to $2^n$ things, but we can only count to $2^n$ - 1*

# Binary Numbers

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To represent numbers, we use Binary (Base 2) numbers

Written using positional notation, similar to decimal numbers:

**Base 10 (Decimal)**

$7309.5_{10} = 7x10^{3} + 3x10^{2} + 0x10^{1} + 9x10^{0} + 5x10^{-1}$

**Base 2 (Binary)**

$1101.101_{2} = 1x2^{3} + 1x2^{2} + 0x2^{1} + 1x2^{0} + 1x2^{-1} + 0x2^{-2} + 1x2^{-3}$

**Base r**

$abc.de = axr^{2} + bxr^{1} + cxr^{0} + dxr^{-1} + exr^{-2}$

Note: Another name for "base" is "radix"

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# Converting from binary to decimal

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**The positional notation gives an easy way to convert from a binary number to the decimal equivalent.**

$1101.101_{2} = 1x2^{3} + 1x2^{2} + 0x2^{1} + 1x2^{0} + 1x2^{-1} + 0x2^{-2} + 1x2^{-3}$

$= 8 + 4 + 0 + 1 + 0.5 + 0 + 0.125$

$= 13.625_{10}$

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# Powers of 2

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Some key values (in decimal):

$2^0$ = 1
$2^1$ = 2
$2^2$ = 4
$2^3$ = 8
$2^4$ = 16
$2^5$ = 32
$2^6$ = 64
$2^7$ = 128
$2^8$ = 256
$2^9$ = 512
$2^{10}$ = 1024 = "1 K"

$2^{20}$ = "1 M"

$2^{30}$ = "1 G"

Know these!

Be careful:

| common computer terminology | common scientific terminology |
|---|---|
| "1K" = $2^{10}$ = 1,024 | "1K" = $10^3$ = 1,000 |
| "1M" = $2^{20}$ = 1,048,576 | "1M" = $10^6$ = 1,000,000 |
| "1G" = $2^{30}$ | "1G" = $10^9$ |

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# Binary Arithmetic

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# Binary addition

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Addition works exactly the same way as in decimal

```
c2   c1
     a1   a0
     b1   b0
-----------
s2   s1   s0
```

Example:

```
1   1 1 1 1
  1 0 1 1 0 1
+ 1 0 0 1 1 1
-------------
1 0 1 0 1 0 0
```

**check:**

$101101 = 2^5+2^3+2^2+2^0 = 32+8+4+1 = 45_{10}$

$100111 = 2^5+2^2+2^1+2^0 = 32+4+2+1 = 39_{10}$

-------                                   ---

$1010100 = 2^6+2^4+2^2 = 64+16+4 = 84_{10}$

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# Binary subtraction

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Subtraction also works the same way as in decimal

```
                 10
           0    ~0~   10
  1   0   ~1~   ~1~   ~0~   1
- 1   0    0     1     1    1
------------------------------
  0   0    0     1     1    0
```

**check:**

$101101 = 45_{10}$

$100111 = 39_{10}$

--------   ---

$110 = 6_{10}$

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# Binary multiplication

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Same method, but much simpler than decimal multiplication because you only ever multiply by “1” to get the interim results

Starting at the right end of the multiplier (bottom number), examine each bit.
- If 0, skip
- if 1, copy the multiplicand right-aligned under the multipier bit you are examining

After all multiplier bits examined, add the interim results

```
            1 0 1 1
         x  1 0 1
            -------
copy        1 0 1 1     copy
        1 0 1 1
        -----------
        1 1 0 1 1 1
```



# Binary Division

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Just like decimal division

```
               0 1 1 1
          ______________
1 1 1   |  1 1 0 1 0 1
        | - 0 0 0
           1 1 0 1
        -    1 1 1
             1 1 0 0
           - 1 1 1
               1 0 1 1
             - 1 1 1
                 1 0 0 (remainder)
```

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# A shortcut: Multiplying/Dividing by the radix

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For a number in base "r",
  shifting left 1 place multiplies by r
  shift right one place divides by r

Decimal:
  503 shifted left = 5030.0
  503 shifted right = 50.3
  500 shifted right three times = 0.503 (= 503 / $10^3$)

Binary:
  1101 shifted left = 11010 ($13_{10}$ x 2 = $26_{10}$)
  1101 shifted right = 110.1 ($13_{10}$ / 2 = $6.5_{10}$)

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# Octal and Hexadecimal

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# Base 8 (Octal)

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| Octal | Decimal Equivalent |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |
| 4 | 4 |
| 5 | 5 |
| 6 | 6 |
| 7 | 7 |
| 10 | 8 |
| 11 | 9 |
| 12 | 10 |
| 13 | 11 |
| 14 | 12 |
| 15 | 13 |
| 16 | 14 |
| 17 | 15 |
| 20 | 16 |

Example:

$147_8 = 1 \times 8^2 + 4 \times 8^1 + 7 \times 8^0$
$= 64 + 32 + 7$
$= 103_{10}$

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# Base 16 (Hexadecimal or “Hex”)

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| Hex Digit | Decimal Equivalent |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |
| 4 | 4 |
| 5 | 5 |
| 6 | 6 |
| 7 | 7 |
| 8 | 8 |
| 9 | 9 |
| A | 10 |
| B | 11 |
| C | 12 |
| D | 13 |
| E | 14 |
| F | 15 |

Examples:

$21_{16} = 2 \times 16^1 + 1 \times 16^0 = 33_{10}$

$5F_{16} = 5 \times 16^1 + 15 \times 16^0 = 95_{10}$

$B65F_{16} = 11 \times 16^3 + 6 \times 16^2 + 5 \times 16^1 + 15 \times 16^0$
$= 45{,}056 + 1{,}536 + 80 + 15$
$= 46{,}687_{10}$

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# Converting between bases

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# Converting from base "r" to Decimal

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- Use the power series expansion:

$$N = (d_2\ d_1\ d_0 \mathbf{.}\ d_{-1}\ d_{-2})_r$$
$$= (d_2 \times r^2 + d_1 \times r^1 + d_0 \times r^0 \mathbf{.}\ d_{-1} \times r^{-1} + d_{-2} \times r^{-2})_{10}$$

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# Converting Integers from Decimal to Base "r"

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For integers, use the "radix divide" technique:

To convert a decimal integer N to base "r",

1. Divide N by r, recording the remainder.
2. If the quotient from the previous step >0, divide it by r, record the remainder and repeat.
3. Write down the remainders from last to first.

$245_{10} = ?_2$

```
2 |245
2 |122   1 ^
 2 | 61  0 |
 2 | 30  1 |
 2 | 15  0 |   = 1 1 1 1  0 1 0 1_2
  2|  7  1 |
  2|  3  1 |
  2|  1  1 |
      0  1 |
```



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# Converting Fractions from Decimal to Base "r"

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For fractions, use the "radix multiply" technique:

To convert a decimal fraction f to base "r",

1. Multiply f by r. Record the integer part of the result.
2. If the fractional part of the result from the previous step =0 or you have the desired number of digits, stop. Else, multiply the fractional part by r, record the integer part and repeat.
3. Write down the integer digits from first to last.

$0.345_{10} = ?_2$ (5 fractional bits)

```
                        recorded integer part
0.345 x 2 = 0.690   0 |
0.690 x 2 = 1.380   1 |
0.380 x 2 = 0.760   0 |   = 0 . 0 1 0 1 1_2
0.760 x 2 = 1.520   1 |
0.520 x 2 = 1.040   1 v
```

*This is 0.34375, not 0.345. The difference is caused by the roundoff error resulting from only using 5 bits to represent the fraction.*

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## Converting mixed numbers from decimal to Base "r"

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Use the Radix Divide technique for the integer part, Radix Multiply for the fractional part.

Example:

From previous 2 slides

$245.345_{10} = 11110101.01011_2$

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## Converting between arbitrary bases

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To determine: $N_p = ?_q$

Use Power Series Expansion to convert N to decimal, then use Radix Multiply/Divide to convert to base q.

$43_5 = ?_3$

$43_5 = 4x5^1 + 3x5^0 = 23_{10}$

```
3 |23
 3 |7   2  ↑
 3 |2   1  |    = 212_3
    0   2  |
```

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# A shortcut for Binary, Octal and Hex



A single digit in base $2^p$ expands to its equivalent p bits in binary.
This lets us convert directly between binary, hex, and octal

$553.76_8 = ?_2$

5 5 3 . 7 $6_8$

101 101 011 . 111 $110_2$

$C6B.2_{16} = ?_2$

C 6 B . $2_{16}$

1100 0110 1011 . $0010_2$

$553.76_8 = ?_{16}$

5 5 3 . 7 $6_8$

000101 101 011 . 111 11000

1 6 B . F $8_{16}$

*Count groups outward from the radix point, not from the left and right end*







# Other Notational Systems

## Signed Magnitude

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Notation: "$N_{sm}$"

The first bit shows the sign of N, "0" for positive, "1" for negative.

The rest of the bits are |N|.

Example:

$+(1101)_2 = (0\ 1101)_{sm}$

$-(1101)_2 = (1\ 1101)_{sm}$

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## 1's Complement

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Notation: $N_{1s}$

The first bit shows the sign of N, as in Signed Magnitude.

If the number is positive, the rest of the bits are |N|.

If the number is negative, the rest of the bits are $(2^n - 1) - |N|$ where n is the # of bits in |N|.

Example:

$+(1101)_2 = (0\ 1101)_{1s}$

$-(1101)_2$
we determine $(2^4 – 1) - 1101 = 1111 - 1101 = 0010$
then $-(1101)_2 = (1\ 0010)_{1s}$

*Note that we can obtain $(2^n - 1) - |N|$ by replacing every "1" in |N| with "0" and every "0" with "1". This is called taking the bit-by-bit complement of |N|.*

*General concept: The complement of 0 is 1, the complement of 1 is 0.*

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## 2's Complement

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Notation: $N_{2s}$

The first bit shows the sign of N, as in Signed Magnitude.

If the number is positive, the rest of the bits are |N|.

If the number is negative, the rest of the bits are $(2^n - 1) - |N| + 1$ where n is the # bits in |N|.

Examples:

$+(1101)_2 = (0\ 1101)_{2s}$

$-(1101)_2$ → 1111 - 1101 + 1 = 0011 → $-(1101)_2 = (1\ 0011)_{1s}$

*Two shortcuts to determine $(2^n-1) - |N| + 1$:*

*(1) Take the bit-by-bit complement of |N| and add 1*
*(2) Start at the right end of |N|, copy all 0's moving left. Copy the first 1. Complement all other bits.*

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## Why complement number systems?

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If numbers are represented in 1's or 2's complement number systems, numbers of either sign will add correctly.

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# Arithmetic using 2's complement

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To add two numbers in 2's complement, add them (including the sign bits) as if they are normal binary numbers and ignore the carry, if any. Surprisingly, the sign bit will always be correct, unless we overflow (see below).

$$\begin{array}{rr} 5_{10} & 0\ 0101_{2s} \\ +4_{10} & 0\ 0100_{2s} \\ \hline 9_{10} & 0\ 1001_{2s} \end{array} \qquad \begin{array}{rr} 5_{10} & 0\ 0101_{2s} \\ +\ -4_{10} & 1\ 1100_{2s} \\ \hline 1_{10} & \not{1}0\ 0001_{2s} \end{array}$$

$$\begin{array}{rr} -5_{10} & 1\ 1011_{2s} \\ +\ 4_{10} & 0\ 0100_{2s} \\ \hline -1_{10} & 1\ 1111_{2s} \end{array} \qquad \begin{array}{rr} -5_{10} & 1\ 1011_{2s} \\ +\ -4_{10} & 1\ 1100_{2s} \\ \hline -9_{10} & \not{1}1\ 0111_{2s} \end{array}$$

$$\begin{array}{rr} 15_{10} & 0\ 1111_{2s} \\ +\ 15_{10} & 0\ 1111_{2s} \\ \hline 30_{10} & 1\ 1110_{2s} \end{array}$$

This is actually $-2_{10}$, not $+30_{10}$. The problem is that we cannot represent 30 with only 4 magnitude bits, so the addition "overflowed". <u>When you add two 2's complement numbers of the same sign and you get the opposite sign for the result</u>, you have an overflow condition.



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# Arithmetic using 1's complement

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To add two numbers in 1's complement, add them (including the sign bits) and if there is a carry, add it into the least-significant bit (the bit at the right end).

$$\begin{array}{rr} 5_{10} & 0\ 0101_{1s} \\ +4_{10} & 0\ 0100_{1s} \\ \hline 9_{10} & 0\ 1001_{1s} \end{array} \qquad \begin{array}{rr} 5_{10} & 0\ 0101_{1s} \\ +\ -4_{10} & 1\ 1011_{1s} \\ \hline 1_{10} & 10\ 0000 \\ & \rightarrow 1 \\ \hline & 0\ 0001_{1s} \end{array}$$

$$\begin{array}{rr} -5_{10} & 1\ 1010_{1s} \\ +\ 4_{10} & 0\ 0100_{1s} \\ \hline -1_{10} & 1\ 1110_{1s} \end{array} \qquad \begin{array}{rr} -5_{10} & 1\ 1010_{1s} \\ +\ -4_{10} & 1\ 1011_{1s} \\ \hline -9_{10} & 11\ 0101 \\ & \rightarrow 1 \\ \hline & 1\ 0110_{1s} \end{array}$$

Overflow is defined the same as in 2's Complement

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# Other Binary Codes

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# Binary Coded Decimal (BCD)

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A way of representing decimal numbers in digital logic

Each digit of the decimal number is represented by 4 bits

| Digit | BCD code |
|---|---|
| 0 | 0000 |
| 1 | 0001 |
| 2 | 0010 |
| 3 | 0011 |
| 4 | 0100 |
| 5 | 0101 |
| 6 | 0110 |
| 7 | 0111 |
| 8 | 1000 |
| 9 | 1001 |

Example:

$134_{10} = (0001\ 0011\ 0100)_{BCD}$

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# BCD Addition

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To add 2 multiple-digit BCD numbers, first add them as if they were normal binary numbers.
Then check each group of 4 bits. If the group is 9 ('1001') or less, it is OK.
When a group of 4 bits is 10 or more, add 6, adding any carry into the next group of 4 bits.

**Example:**

| Decimal | BCD |
|---|---|
| 184 | 0001 1000 0100 |
| + 576 | 0101 0111 0110 |
| | 0110 1111 1010 |
| | 0110 0110 |
| 760 | 0111 0110 0000 |

These two groups of 4 bits are >9, so we have to adjust by adding 6 to each of them.

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# Excess-3 code

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Another way of representing decimal numbers.

Each digit of the decimal number is represented by 4 bits, equal to the BCD code plus 3.

| Digit | Excess-3 code |
|---|---|
| 0 | 0011 |
| 1 | 0100 |
| 2 | 0101 |
| 3 | 0110 |
| 4 | 0111 |
| 5 | 1000 |
| 6 | 1001 |
| 7 | 1010 |
| 8 | 1011 |
| 9 | 1100 |

Example:

$134_{10} = (0100\ 0110\ 0111)_{Excess\text{-}3}$

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# Gray Code

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Designed so that only one bit changes when going between two consecutive numbers

| Decimal | Gray code |
|---|---|
| 0 | 0000 |
| 1 | 0001 |
| 2 | 0011 |
| 3 | 0010 |
| 4 | 0110 |
| 5 | 0111 |
| 6 | 0101 |
| 7 | 0100 |
| 8 | 1100 |
| 9 | 1101 |
| 10 | 1111 |
| 11 | 1110 |
| 12 | 1010 |
| 13 | 1011 |
| 14 | 1001 |
| 15 | 1000 |

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# Alphanumeric codes

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Used to represent alphabetical, number, punctuation, symbols in digital systems (also some common control signals)

Most widely used: American Standard Code for Information Exchange (ASCII)

See text page 24 for list.

Some examples:

| Symbol | ASCII code |
|---|---|
| "A" | 100 0001 |
| "a" | 110 0001 |
| "2" | 011 0010 |
| "!" | 010 0001 |
| "line feed" | 000 1010 |
| "delete" | 111 1111 |

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## Error-detecting codes

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Designed to allow detection of some types of errors in digital data.

Example:

Odd parity: add a bit to the data item to make the total number of 1's odd

ASCII "A" = 100 0001

with odd parity bit: 1100 0001

We will talk about error correction codes later this term.

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# The Basic Logic Operations

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**AND** x·y or xy

| x | y | xy |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

*This is a "Truth Table" (The operation's result for every combination of values)*

**OR** x + y

| x | y | x+y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

**NOT** or **COMPLEMENT** or **INVERT** x'

| x | x' |
|---|---|
| 0 | 1 |
| 1 | 0 |

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# Using Logic Expressions

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We can write logical and arithmetic expressions by combining logic values, variables, and operations

Example:
"If it's Winter, the grass is brown. If it's Summer and it's a drought, the grass is brown. Else, the grass is green."

Define variables:

S=0 if it's Winter, S=1 if it's Summer
D=1 if there's a drought, D=0 if there's no drought.
B=1 if the grass is brown, B=0 if the grass is green

Truth Table:

| S | D | B |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

⇨

Logic Expression:

B = S'D' + S'D + SD

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# Using Logic Expressions (2)

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Example (arithmetic):

```
   a
+  b
 c s
```

⇨

Truth Table:

| a | b | c | s |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Logic Expressions:

c = ab

s = a'b + ab'



# Digital Logic Circuits

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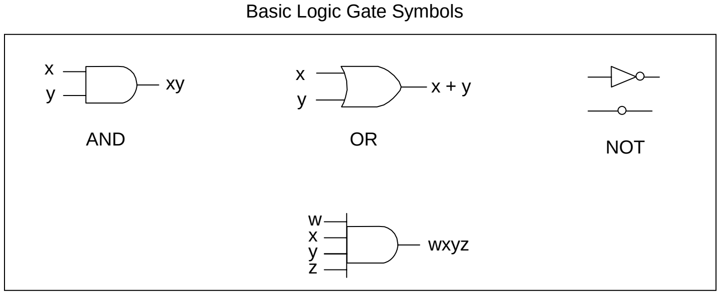
Implementation of Logic variables and operations in electronics

- Values (0 and 1) represented by voltage levels
- Variables implemented by electrical connections
- Operations implemented by logic gates

Basic Logic Gate Symbols

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