Logic, the world and theory

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Big Picture

Expressiveness

• The world is all that exists, even the unexpressed.
• The sum of all languages and language forms is all that can be expressed.
• The representation of things that are true or false or true and not-true is all that logic can represent.
• The consistent fusion of language and logic produces all theory.
• The work space for any problem is constrained by all.
Control

- The world cannot be controlled.
- All languages taken together are controlled by the speakers and their history.
- All logics are controlled by consistency.
- All theories are controlled by the world, logic and language.
- The work space is controlled by all.

Initial Assumptions

- Logic considers only two values; true and false.
- Only declarative and “literal” sentences of language can be either true or false.
- The internal representations in a sentence may or may not be important and in either case only the truth value is important.

Expanding Logic

- Propositional logic: the internal structure of the proposition is ignored and only propositional connectives are relevant.
- First order logic: adding to propositional logic objects, predicates and quantifiers over objects. (Higher orders are possible.)
- Modal logic: at least collections of propositions in which propositional logic holds and accessibility relations between the collections.
Computers transform data

• Logic Models

• Well-defined transformation rules preserve and/or derive symbols and relations in the model

Interpreting a model

• A model is just symbols

Examples of interpretations

• Suppose we are presented with a simple model containing the following two relations:
  \[ \text{parent}(\text{Edward}, \text{George}) \]
  \[ \text{parent}(\text{George}, \text{Edward}) \]

• Many conclusions are possible
  - E.g., language and culture of the example are English.
  - The parent relation denotes the relationship between a human parent and their child.
  - Edward and George are English human names, representing actual humans.
  - If we know British history, these are important historical figures; namely, British monarchs.
More interpretations

- 2 possible interpretations
  - the first symbol represents the parent of the second, or
  - the second symbol represents the parent of the first.
- Assume order is consistent between the two.

More interpretations

- Assume that both of the relations given are true; i.e., hold at the same time.
  - An apparent inconsistency
    - A person Edward who is the parent of a person George cannot simultaneously have that same person George as its parent
    - Violates what we know about the parent relationship between humans.
  - We therefore would assume that the knowledge model includes at least three distinct persons, with two of them named Edward, or two of them named George.

Still more interpretations!

- Names not chosen at random
  - Chosen to exploit our knowledge in clarifying the example
    - Note that for those unfamiliar with British monarchs, this social/cultural cue does not occur.
  - Using previous two assumptions, we infer that arguments are parent then child, since for British monarchs, only George V had someone named Edward as both his parent or child.
  - Infer that the individuals denoted are as follows:
    - parent( Edward-VII-British-monarch, George-V-British-monarch )
    - parent( George-V-British-monarch, Edward-VIII-British-monarch )
  - We could also conclude an additional relationship:
    - grandparent( Edward-VII-British-monarch, Edward-VIII-British-monarch )