CS309 Spring 2006 Midterm Exam 1 NAME___ANSWERS_____

Show all work clearly – if I can't follow your reasoning or decipher your writing, I will not give you credit.

Proofs and simplifications must indicate the Postulate or Theorem number used for each step.

Write all answers on the exam sheet. Use backs of pages as needed. If you need more paper, ask.

No calculators, PDAs, etc.

Each numbered question is worth 10 points.

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1. Perform the indicated base conversions. Unless stated otherwise, you may use shortcut techniques

   a. $(110100.11)_2 = (?)_{10}$

   $= 2^5 + 2^4 + 2^2 + 2^{-1} + 2^{-2}$
   $= (52.75)_{10}$

   b. $(A4)_{16} = (?)_{10}$

   $= 10 \times 16 + 4 \times 1$
   $= (164)_{10}$

   c. $(1101000.11)_2 = (?)_{16}$

   $= (0110\ 1000\ .\ 1100)$
   $= (\ 6\ \ 8\ .\ \ C\ )_{16}$

   d. $(13.625)_{10} = (?)_2$ (use the Radix divide/multiply technique)

```
2 | 13      ------      .625
  |  |      |    |      x   2
2 |  6  1   |    |      1.250
  |  |      |    |      x   2
2 |  3  0   |    |      0.500
  |  |      |    |      x   2
2 |  1  1   |   \ /     1.000
  |         |
    0  1    |
```

   $= (1101.101)_2$

   e. $(126)_8 = (?)_{16}$

$$
\begin{aligned}
&(1\ \ \ 2\ \ \ 6\ )_8 \\
&=(001\ \ 010\ \ 110\ )_2 \\
&=(0000\ 0101\ 0110)_2 \\
&=(0\ \ \ 5\ \ \ 6\ \ \ )_{16}
\end{aligned}
$$

2. Perform the following arithmetic:

   a. Use Binary arithmetic to determine $(1110.01)_2 + (1011)_2 = (?)_2$

      $= (11001.01)_2$

   b. Use Binary arithmetic to determine $(11000)_2 - (10)_2 = (?)_2$

      $=(10110)_2$

   c. In binary, multiply $(1011101)_2$ by $(100)_2$. Show the result in binary. You may use shortcut techniques.

      Shifting left 2 places yields $(101110100)_2$

   d. Use BCD arithmetic to determine $(49)_{10} + (19)_{10} = (?)_{BCD}$

$$\begin{array}{l} (0100\ 1001)_{BCD} \\ +(0001\ 1001)_{BCD} \\ \quad 0110\ 0010 \\ +\quad\ \ 0110 \\ (0110\ 1000)_{BCD} \end{array}$$

      Note: In this case, we add 6 because there was a carry from the low-order BCD digit. This is the same as the "invalid BCD digit" we discussed in class. See p18 of the text. This problem was counted as extra credit.

3. Perform the following arithmetic. Show all work including sign bits.

   a. Use the 2's Complement Number System with 4 magnitude bits to determine: $(5)_{10} + (-12)_{10} = (?)_{2's}$.

$$(5)_{10} = (0,0101)_{2's}$$
$$(-12)_{10} = (1,0100)_{2's}$$

$$\begin{array}{l} \quad 0,0101 \\ +1,0100 \\ (1,1001)_{2's} \end{array}$$

   b. Use the 2's Complement Number System with 4 magnitude bits to determine $(-5)_{10} - (12)_{10} = (?)_{2's}$

      (-5)10 = (1,1011)2's
      (-12)10 = (1,0100)2's

$$\begin{array}{l} \quad 1,1011 \\ +1,0100 \\ (10,1111) \end{array}$$

      overflow, because we added two negatives and got a positive.

c. Use the 1's Complement Number System with 4 magnitude bits to determine: $(-5)_{10} + (-7)_{10} = (?)_{1's}$

$(-5)_{10} = (1,1010)_{1's}$
$(-7)_{10} = (1,1000)_{1's}$

```
  1,1010
 +1,1000
 11,0010
 |
  ----->1
 (1,0011)1's
```

d. Use the 1's Complement Number System with 4 magnitude bits to determine: $(-5)_{10} + (12)_{10} = (?)_{1's}$

$(-5)_{10} = (1,1010)_{1's}$
$(12)_{10} = (0,1100)_{1's}$

```
  1,1010
 +0,1100
 10,0110
 |
  ----->1
 (0,0111)1's
```

4. Draw the logic symbol, write the Truth Table, and give the Boolean expression for the following gate types:

**** For answers, see text or class notes*****

a. 2-input OR

b. 3-input NAND

c. 2-input XOR

d. 2-input NOR

e. NOT

5. For the Boolean expression f(x,y,z) = x'y + xz(y + z),

a. Write the Truth Table

| xyz | x'y | xz | y+z | f(x,y,z) |
|---|---|---|---|---|
| 000 | 0 | 0 | 0 | 0 |
| 001 | 0 | 0 | 1 | 0 |
| 010 | 1 | 0 | 1 | 1 |
| 011 | 1 | 0 | 1 | 1 |
| 100 | 0 | 0 | 0 | 0 |
| 101 | 0 | 1 | 1 | 1 |
| 110 | 0 | 0 | 1 | 0 |
| 111 | 0 | 1 | 1 | 1 |

b. Write the expression in Minterm List form

From the truth table,
f(x,y,z) = Sum of Minterms (2,3,5,7)

c. Write the expression in Maxterm List form

From the truth table
f(x,y,z) = Product of Maxterms (0,1,4,6)

d. Write expression in Canonical SOP form.

f(x,y,z) = x'yz' + x'yz + xy'z + xyz

6. Using only the definitions, Postulates and Theorems given in Table 1, simplify the Boolean expressions:

a. k(A,B,C) = A'C' + ABC + AC'

| | |
|---|---|
| A'C' + ABC +AC' | |
| = A'C' + AC' +ABC | P3A |
| = C'A' + C'A +ABC | P3B |
| = C'(A'+A) + ABC | P4A |
| = C'(1) + ABC | P5A |
| = C' +ABC | P2A |
| = (C'+AB)(C'+C) | P4B |
| = (C'+AB)(1) | P5A |
| = C'+AB | P2A |

b. j(A,B,C,D) = A'B(D' + C'D) + B(A + A'CD)

| | |
|---|---|
| A'B(D' + C'D) + B(A + A'CD) | |
| = A'BD' + A'BC'D + BA + BA'CD | P4A |
| = A'BD' + AB + A'BDC + A'BDC' | P3A, P3B |
| = A'BD' + AB + A'BD(C + C') | P4A |
| = A'BD' + AB + A'BD(1) | P5A |
| = A'BD' + AB +A'BD | P2B |
| = A'BD' + A'BD +AB | P3A |
| = A'B(D'+D) +AB | P4A |
| = A'B(1) +AB | P5A |
| = A'B + AB | P2B |
| = BA' + BA | P3B |
| = B(A'+A) | P4A |
| = B(1) | P5A |
| = B | P2B |

7. Complement and dual

   a. Determine an SOP-form Boolean expression for the complement of p(x,y,z) = xy'(y+z) + x'.

   "*" indicates an acceptable answer

   p'(x,y,z) = (xy'(y+z)+x')'

| | |
|---|---|
| = (xy'(y+z))' x'' | deMorgan |
| = (xy'(y+z))' x | T3 |
| = ( (xy')' + (y+z)' ) x | deMorgan |
| = ( (x'+y'')+ (y'z') ) x | deMorgan |
| = ( (x'+y) + y'z') x | T3 |
| = x(x'+y) + xy'z' | P4A |
| = xx' + xy + xy'z' | P4A * |
| = 0 + xy + xy'z' | P5B * |
| = xy +xy'z' | P2A * |
| = x(y + y'z') | P4A |
| = x (y + y')(y + z') | P4B |
| = x (1)(y + z') | P5A |
| = x (y + z') | P2B |
| = xy + xz' | P4A * |

   ANOTHER WAY:
   From the truth table, p(x,y,z) = Sum of Minterms (0-3,5)
   So, p'(x,y,z) = Sum of Minterms (4,6,7)
   = xy'z' + xyz' + xyz *

   b. Write the dual of the expression, x + x'y = 0 + x + y.

   x(x'+y) = 1xy

8. For the circuit in Figure 1, write the Boolean expression for z(A,B,C). Do not simplify the result.

   Z = AB' + (BC)'

9. Draw a logic diagram using AND, OR, and NOT gates for the Boolean expression Z=(A+B')(A+C')(B+C'). You are not required to simplify the expression.

   Presented in class

10. Draw a logic diagram using NAND gates for the Boolean expression Y=A'B + BC'+B'C. You are not required to simplify the expression.

    Presented in class

TABLE 1

| | | |
|---|---|---|
| P2 identity | (a) x+0=x | (b) x AND 1=x |
| P3 commutative | (a) x+y = y+x | (b) xy = yx |
| P4 distributive | (a) x(y+z) = xy+xz | (b) x+yz = (x+y)(x+z) |
| P5 Def of complement | (a) x+x’ = 1 | (b) xx’ = 0 |
| | | |
| T1 | (a) x+x=x | (b) xx=x |
| T2 | (a) x+a=1 | (b) x AND 0 = 0 |
| T3 involution | (x’)’ = x | |
| T4 associative | (a) x+(y+z)=(x+y)+z | (b) x(yz)=(xy)z |
| T6 absorption | (a) x+xy = x | (b) x(x+y)=x |

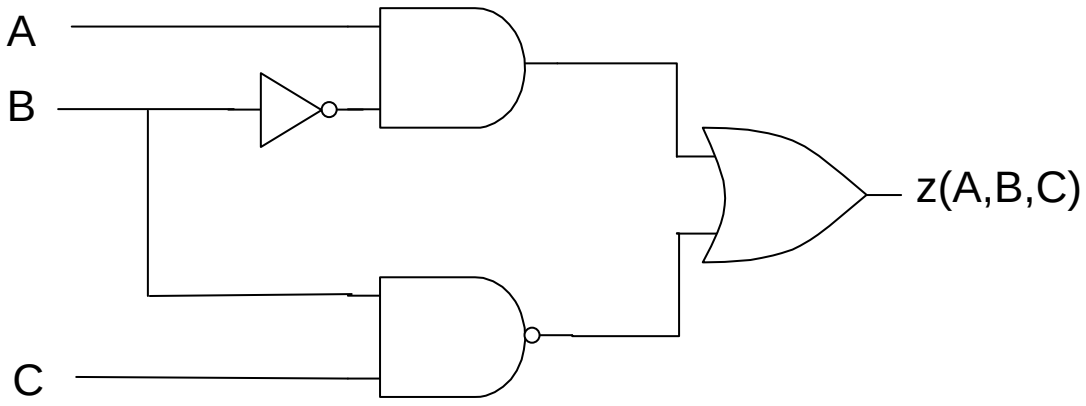


Figure 1